Encrypting Long and Variable-Length Messages

Block Cipher Modes of Operation

CS/ECE 407

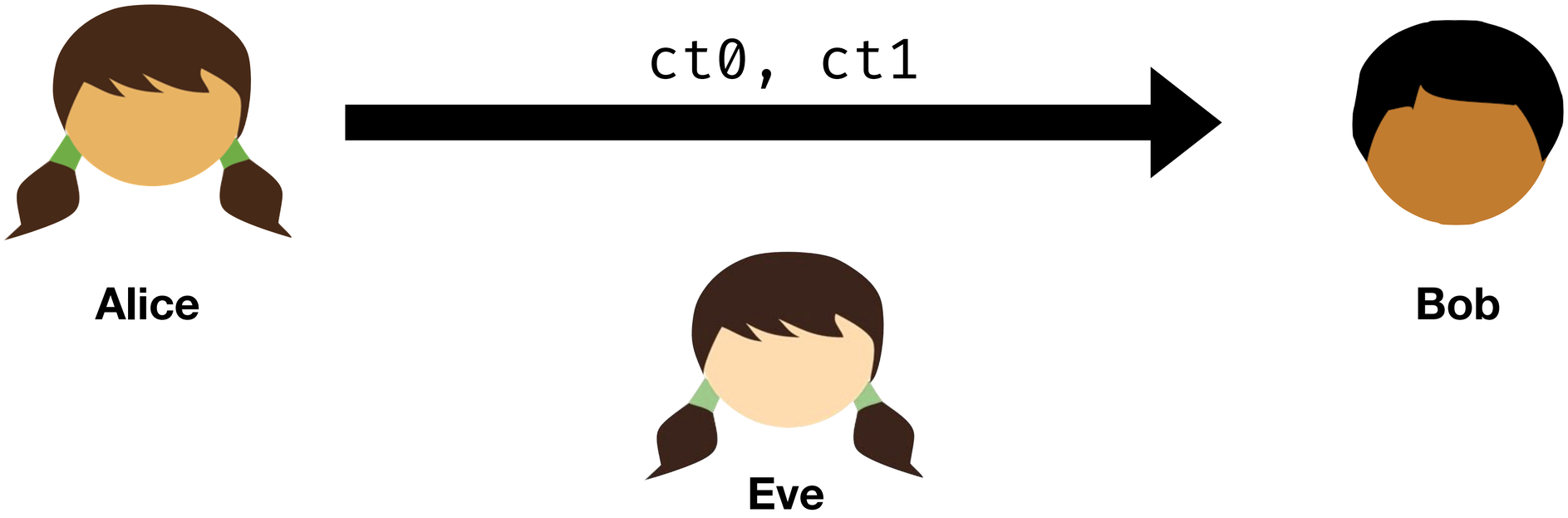
Today's objectives

Discuss Block Cipher Modes of Operation

See how to encrypt long messages

Explain problem of variable length messages

Show how to pad messages to achieve CPA security



A cipher (Enc, Dec) has **security against a chosen plaintext attack (CPA)** if:

```
k ←$ {0,1}^λ
eavesdrop(m0, m1):
  ct ← Enc(k, m0)
  return ct
```

$\overset{c}{\approx}$

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$$F : \{0,1\}^\lambda \times \{0,1\}^n \to \{0,1\}^n$$

F is called a **pseudorandom permutation (or block cipher)** if:

There exists F^{-1} s.t. $F^{-1}(k, F(k,x)) = x$

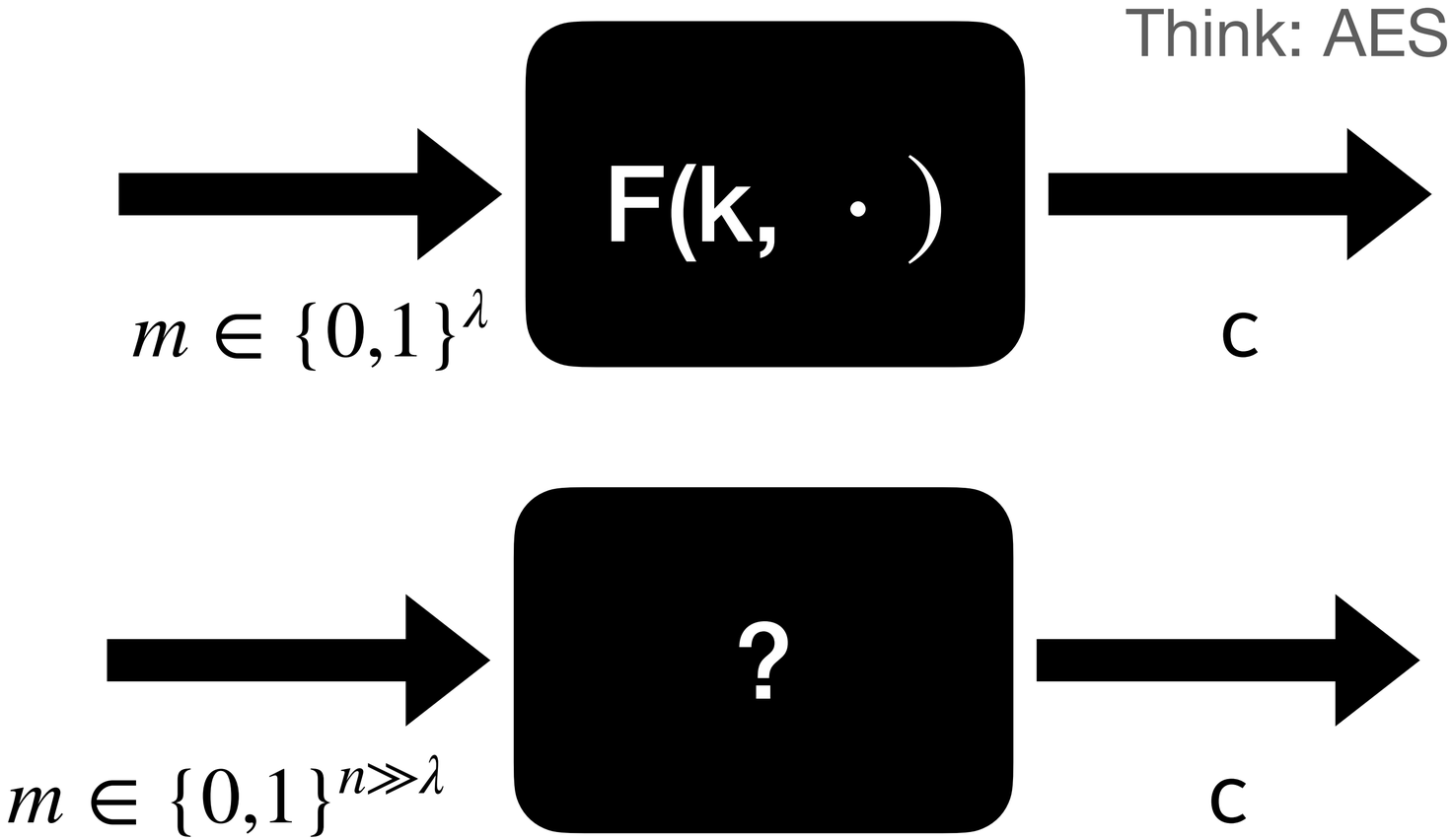
and

```
k ←$ {0,1}^λ

apply(x):
  return F(k, x)
```

$\overset{c}{\approx}$

```
D ← empty-dictionary

apply(x):
  if x is not in D:
    D[x] ←$ {0,1}^n \ values
  return D[x]
```

Block Cipher Modes of Operation



Randomized CPA-Secure Encryption

```
Enc(k, m):
  r ←$ {0,1}^λ
  c0 ← F(k, r) ⊕ m
  c ← (c0, r)
  return c

Dec(k, (c0, r)):
  return F(k, r) ⊕ c0
```

In practice, this doubles the length of ciphertexts!

Problematic for long messages

Can we **amortize** this added cost?

Block Cipher Modes of Operation

Electronic Codebook (ECB) Mode —
WARNING: NOT RECOMMENDED!

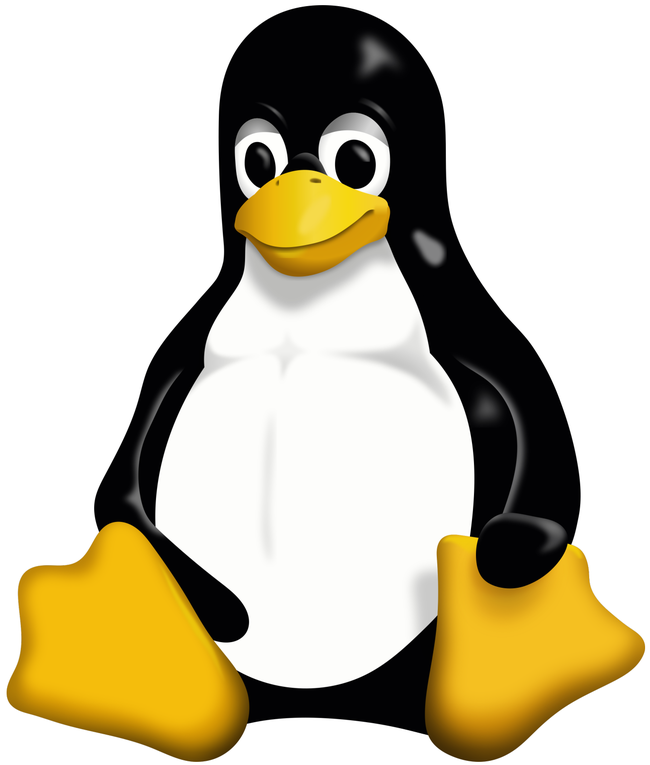
Cipher Block Chaining (CBC) Mode — Very common in practice

Counter (CTR) Mode

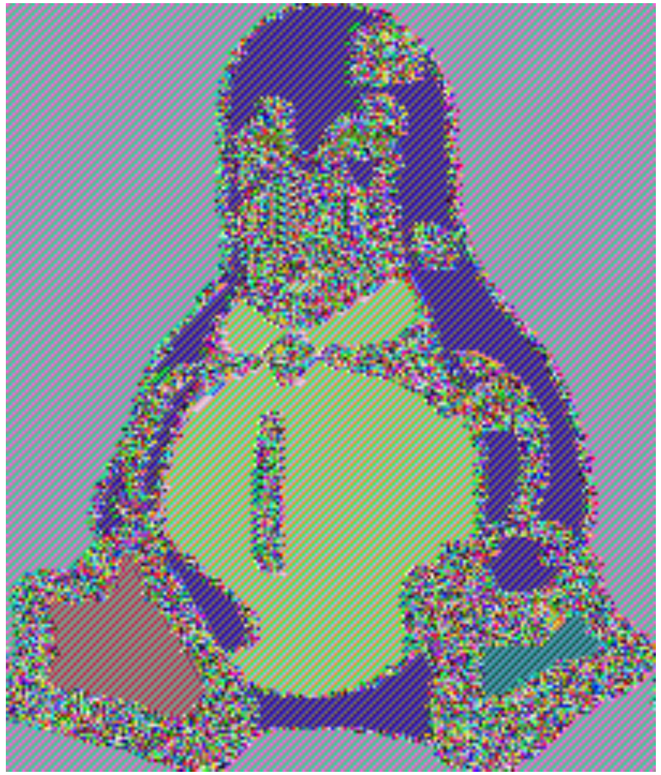
Electronic Codebook (ECB) Mode — WARNING: NOT RECOMMENDED!

```
Enc(k, m_1 | … | m_n):
  for i in 1 to n
    c_i ← F(k, m_i)
  return c_1 | … | c_n

Dec(k, c_1 | … | c_n):
  for i in 1 to n
    m_i ← F^{-1}(k, c_i)
  return m_1 | … | m_n
```

ECB Mode: Do not use!!!



“Good” encryption



ECB Mode

Cipher Block Chaining (CBC) Mode

```
Enc(k, m_1 | … | m_n):
  c_0 ←$ {0,1}^λ
  for i in 1 to n
    c_i ← F(k, m_i ⊕ c_i-1)
  return c_0 | c_1 | … | c_n

Dec(k, c_0 | c_1 | … | c_n):
  for i in 1 to n
    m_i ← F^-1(k, c_i) ⊕ c_i-1
  return m_1 | … | m_n
```

“initialization vector”

Counter (CTR) Mode

```
Enc(k, m_1 | … | m_n):
  r ←$ {0,1}^λ
  for i in 1 to n
    c_i ← F(k, r + i) ⊕ m_i
  return r | c_1 | … | c_n

Dec(k, r | c_1 | … | c_n):
  for i in 1 to n
    m_i ← F(k, r + i) ⊕ c_i
return m_1 | … | m_n
```

“initialization vector”

Block Cipher Modes of Operation

Electronic Codebook (ECB) Mode —
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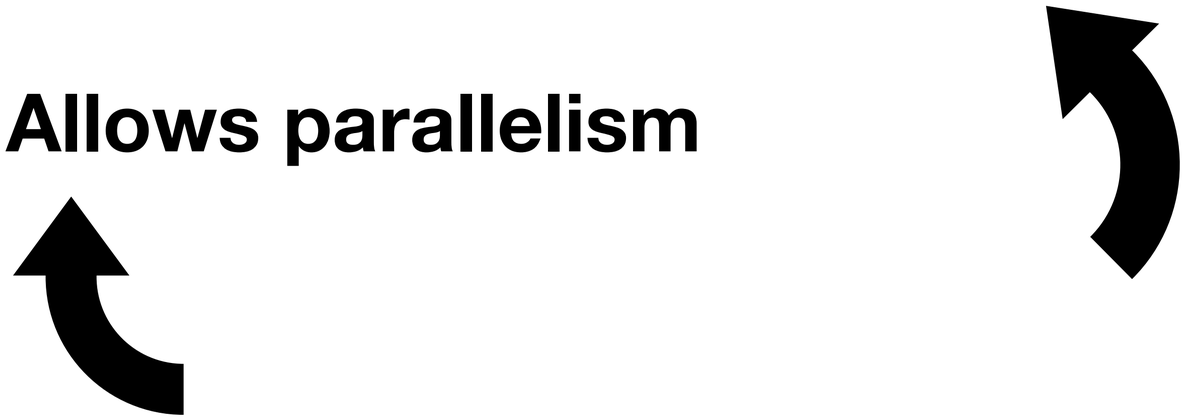
Cipher Block Chaining (CBC) Mode — Very common in practice

Counter (CTR) Mode — Allows parallelism



Can be adjusted to achieve CPA Security

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Definition is too strict! It only works for **fixed-length** messages

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```
k ←$ {0,1}^λ
eavesdrop(m0, m1):
  if |m0| ≠ |m1|:
    return error
  ct ← Enc(k, m0)
  return ct
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  if |m0| ≠ |m1|:
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  ct ← Enc(k, m1)
  return ct
```

Padding:

Consider:

$$\texttt{ct} \leftarrow \texttt{Enc}(\texttt{k}, 0^{\lambda-1})$$

How should we handle this?

Padding:

`pad(m)` : takes input message, outputs string whose length is multiple of block length

`unpad(m)` : inverse of pad

Correctness: `unpad(pad(m)) = m`

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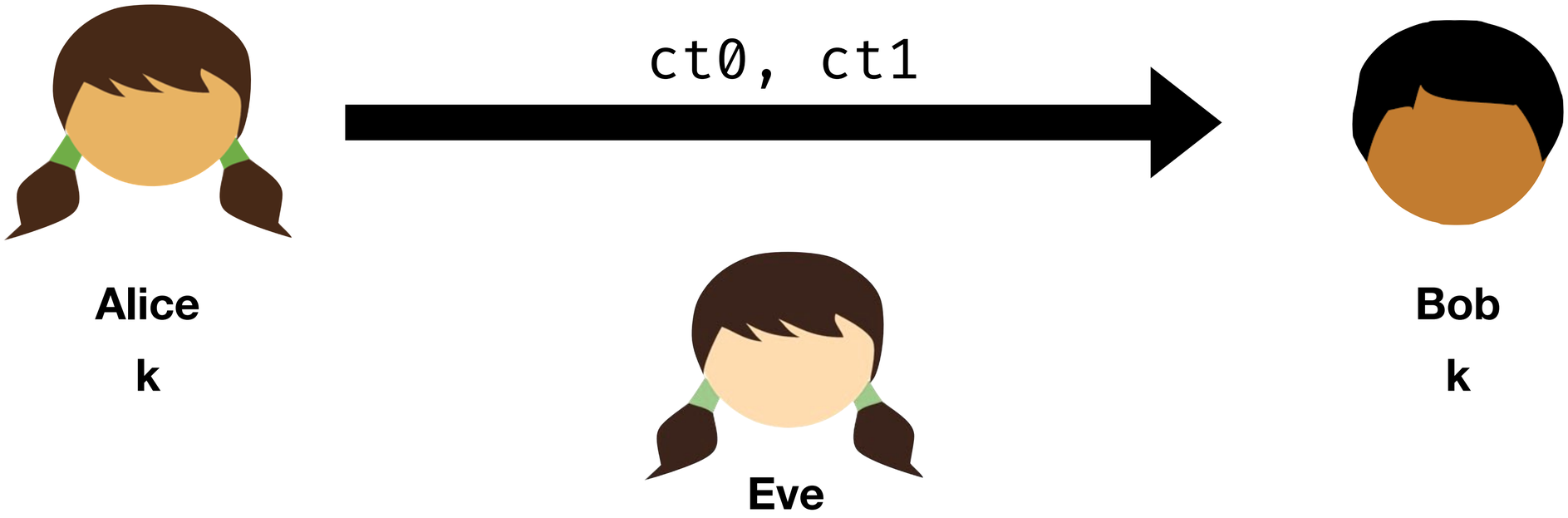
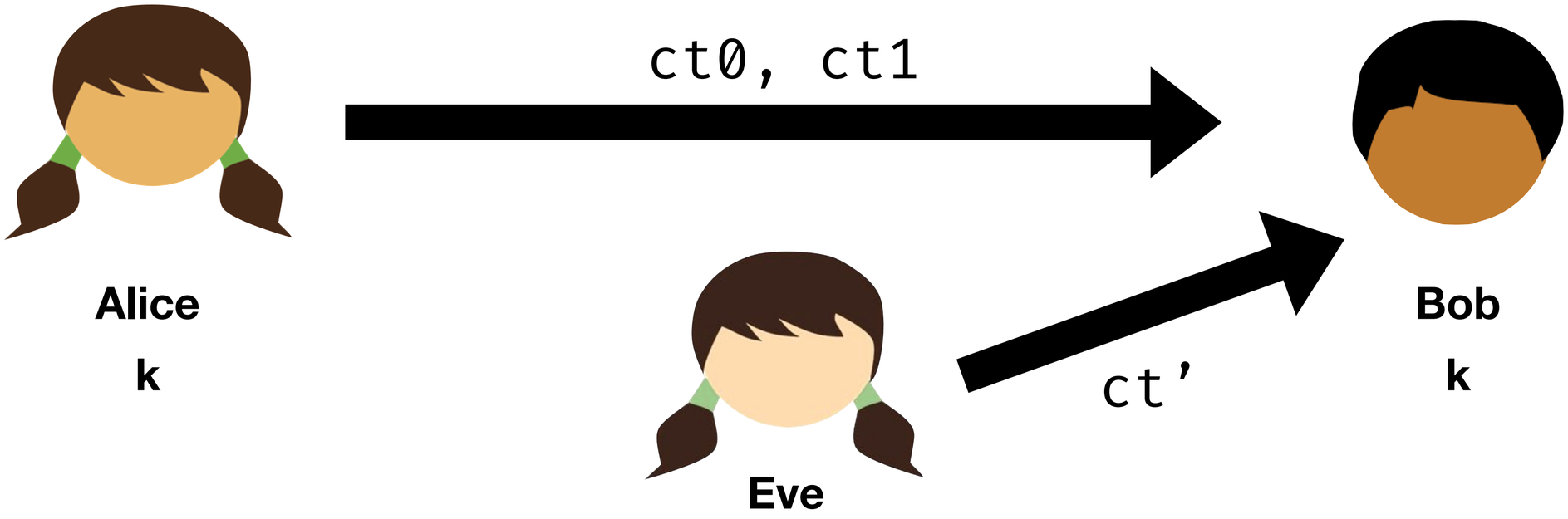
Exercise: suppose that m is already a multiple of the block length.
Does Alice need to pad it?



Alice and Bob can now exchange arbitrary numbers of arbitrary-length messages with confidentiality

However, we have no notion of **authenticity**



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However, we have no notion of **authenticity**

So far our definition of security provides no way for Bob to check that a ciphertext is a “good one"

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